

One-Dimensional Fermions with neither Luttinger-Liquid nor Fermi-Liquid Behavior

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It is well-known that, generically, the one-dimensional interacting fermions cannot be described in terms of the Fermi liquid. Instead, they present different phenomenology, that of the Tomonaga-Luttinger liquid: the Landau quasiparticles are ill-defined, and the fermion occupation number is continuous at the Fermi energy. We demonstrate that suitable fine-tuning of the interaction between fermions can stabilize a peculiar state of one-dimensional matter, which is dissimilar to both the Tomonaga-Luttinger and Fermi liquids. We propose to call this state a quasi-Fermi liquid. Technically speaking, such liquid exists only when the fermion interaction is irrelevant (in the renormalization group sense). The quasi-Fermi liquid exhibits the properties of both the Tomonaga-Luttinger liquid and the Fermi liquid. Similar to the Tomonaga-Luttinger liquid, no finite-momentum quasiparticles are supported by the quasi-Fermi liquid; on the other hand, its fermion occupation number demonstrates finite discontinuity at the Fermi energy, which is a hallmark feature of the Fermi liquid. Possible realization of the quasi-Fermi liquid with the help of cold atoms in an optical trap is discussed.

Introduction.— An important goal of the modern many-body physics is the search for exotic states of matter. Appropriate examples are spin liquids [1–3], Majorana fermion [4–8], topological insulators and semimetals [9–11], and others. A peculiar state of one-dimensional (1D) fermionic matter deviating from known types of interacting Fermi systems is the subject of this paper.

Let us remind ourselves that the most basic model of the interacting fermions is that of the Fermi liquid. It successfully describes a variety of interacting fermion systems (e. g., electrons in solids, atoms of helium-3) [12]. The approach is based on the Landau’s conjecture that both the ground state of a Fermi liquid and its low-lying excitations are adiabatically connected to states of the non-interacting Fermi gas. If the interaction is weak, this hypothesis implies that the perturbation theory in the interaction strength is valid. The latter supplies a theorist with a tool to study specific examples.

A known system for which the Landau conjecture fails is a 1D liquid of interacting fermions. The interacting 1D fermions constitute a separate universality class, so-called Tomonaga-Luttinger liquid [13, 14]: unlike the Fermi liquid, the Tomonaga-Luttinger ground and excited states have zero overlap with the corresponding non-interacting states, the Tomonaga-Luttinger liquid properties cannot be calculated perturbatively with interaction strength as a small parameter.

In 1D the Tomonaga-Luttinger liquid is a generic state of matter. However, recent progress in fabrication and control over the properties of the many-particle systems allows us to look for more fragile types of 1D correlated liquids. Specifically, consider a gas of Fermi atoms in a 1D trap [15]. It is within modern experimental capabilities to vary the effective interaction constant of the optically trapped atoms, and even tune the constant to zero [16, 17]. Below we will demonstrate that such nullification of the *effective* coupling constant does not imply

vanishment of all *microscopic* interactions. Some residual interactions remain, and in 1D they stabilize a peculiar state of matter, which we propose to call a quasi-Fermi liquid. The latter state appears to be a hybrid of both the Fermi and the Tomonaga-Luttinger liquids: its ground state is perturbatively connected to the ground state for the free fermions, yet, the perturbatively-defined quasiparticles do not exist. That is, in case of the quasi-Fermi liquid the Landau conjecture valid only for the ground state, but not for excitations. Of course, there is nothing special about the cold atoms, and the quasi-Fermi liquid may be realized in other fermion systems, which allow adequate fine-tuning of the coupling.

The presentation below has the following structure. First, we formally introduce our model. Second, the self-energy is evaluated perturbatively, which allows to determine both the quasiparticle residue and the occupation number corrections. Third, analyzing these quantities we will be able to define the quasi-Fermi liquid as a distinct state of fermionic matter. Fourth, we discuss possible implementation of such quantum liquid using optically trapped cold atoms. Finally, we formulate our conclusions. In the Supplemental Material we present the extension of our calculations beyond the second-order perturbation theory, and discuss other subtleties.

The studied model.— The 1D interacting fermions are commonly described by the Tomonaga-Luttinger Hamiltonian:

$$H_{\text{TL}} = H_{\text{kin}} + H_{\text{int}}, \quad (1)$$

$$H_{\text{kin}} = iv_F \int dx \left(: \psi_L^\dagger \nabla \psi_L : - : \psi_R^\dagger \nabla \psi_R : \right), \quad (2)$$

$$H_{\text{int}} = g \int dx \rho_L \rho_R, \quad (3)$$

where ψ_p , is the field operator for the right-moving ($p=R$) and left-moving ($p=L$) fermions, operators $\rho_p = : \psi_p^\dagger \psi_p :$ are the densities of the left- and right-movers, v_F is the

Fermi velocity, g is the coupling constant. Colons denote the normal ordering.

The Tomonaga-Luttinger liquid differs from the Fermi liquid: the perturbatively-defined quasiparticles are absent, the Fermi occupation number $n_k^p = \langle c_{pk}^\dagger c_{pk} \rangle$ has no discontinuity at the Fermi point, the Tomonaga-Luttinger ground state has zero overlap with the free fermion ground state.

The culprit responsible for these abnormalities is the fermion-fermion interaction H_{int} , which is marginal in the renormalization group sense. The perturbation theory in orders of g has additional divergences absent in the higher-dimensional systems. For example, the Matsubara single-particle self-energy is equal to [18–20]

$$\Sigma_{\text{TL}}^p = \frac{g^2}{16\pi^2 v_F^2} (i\nu - p v_F k) \ln \left(\frac{v_F^2 k^2 + \nu^2}{4v_F^2 \Lambda^2} \right) + \dots, \quad (4)$$

where the ellipsis stands for the less singular terms, $p = +1$ ($p = -1$) for the right-moving (left-moving) fermions, and Λ is the ultraviolet cutoff. This self-energy corresponds to the following expression

$$\delta Z_{\text{TL}}^p = \frac{g^2}{16\pi^2 v_F^2} \ln \left(\frac{4v_F^2 \Lambda^2}{v_F^2 k^2 + \nu^2} \right) + \dots, \quad (5)$$

for correction to the quasiparticle residue $Z_{\text{TL}}^p = 1 - \delta Z_{\text{TL}}^p$. The correction diverges for small ν and k . As a result, the conventional Fermi quasiparticles are ill-defined, and the occupation number function has a power-law singularity instead of the discontinuity. The properties of H_{TL} , Eq. (1), are now well-understood [13, 14].

However, it is sometimes required to include the irrelevant operators into consideration. There are two least irrelevant operators:

$$H_{\text{nl}} = v_F' \int dx \left[:(\nabla \psi_L^\dagger)(\nabla \psi_L): + :(\nabla \psi_R^\dagger)(\nabla \psi_R): \right], \quad (6)$$

$$H_{\text{int}}' = ig' \int dx \left\{ \rho_R \left[: \psi_L^\dagger (\nabla \psi_L) : - : (\nabla \psi_L^\dagger) \psi_L : \right] - \rho_L \left[: \psi_R^\dagger (\nabla \psi_R) : - : (\nabla \psi_R^\dagger) \psi_R : \right] \right\}. \quad (7)$$

Here H_{nl} is the quadratic correction to the linear dispersion of the fermions, H_{int}' is the irrelevant interaction. Both H_{nl} and H_{int}' have the scaling dimension of 3 (the dimension of the gradient operator is 1, each field operator has the dimension of 1/2). Other irrelevant operators have higher scaling dimensions, therefore, their effects are less pronounced.

Recently, the Hamiltonian

$$H = H_{\text{TL}} + H_{\text{nl}} + H_{\text{int}}' \quad (8)$$

and its modifications have been investigated actively [21–39]. These studies have demonstrated that combined effect of the marginal and the irrelevant operators has important and measurable consequences for system's properties.

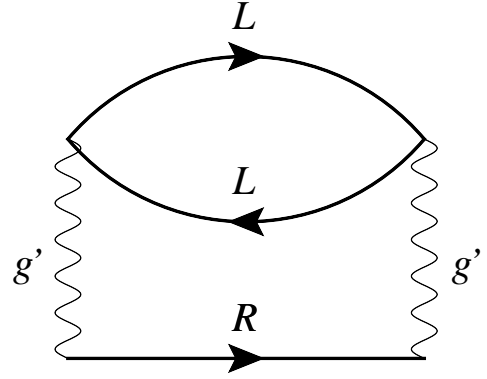


FIG. 1: The leading self-energy correction diagram. The solid lines with arrows and ‘L’, ‘R’ chirality labels correspond to the fermion propagators. The wiggly lines are irrelevant interactions.

In this paper we will discuss the model of the 1D fermions without the marginal interaction at all:

$$H_{\text{ii}} = H_{\text{kin}} + H_{\text{nl}} + H_{\text{int}}', \quad (9)$$

where ‘ii’ stands for ‘irrelevant interaction’. We may name two examples where H_{ii} is applicable. First, consider the cold Fermi atoms in a 1D trap. Under rather general conditions the suitable Hamiltonian is given by Eq. (1), see Refs. [17, 40]. However, the interaction between the atoms is highly adjustable [16, 17], which may be used to our advantage: below we will offer an argument suggesting that the system parameters can be tuned in such a manner that g [or, more precisely, renormalized coupling $g^{\text{eff}} = g + \mathcal{O}((g')^2)$] vanishes, but $g' \neq 0$.

Our second case requires no fine-tuning. Using the unitary transformation of Ref. 41, it has been demonstrated that the Tomonaga-Luttinger Hamiltonian with non-linear dispersion, Eq. (8), may be mapped [23, 25, 33] on Hamiltonian H_{ii} (see also Ref. 42). Therefore, the properties of H_{ii} are important for the theoretical description of the generic model H .

Superficially, one expects that, since H_{ii} has only the irrelevant interaction, it describes a kind of 1D Fermi liquid. Indeed, using the perturbation theory, we will demonstrate that the correction to the fermion occupation number n_k^p is finite and small. However, in a drastic departure from the Fermi liquid picture, the quasiparticle residue correction diverges on the mass surface. Thus, Hamiltonian H_{ii} describes a state of 1D matter which lies halfway between the Fermi liquid and the Tomonaga-Luttinger liquid: n_k^p has the finite discontinuity at the Fermi energy, but no perturbatively defined quasiparticles exist. This is our quasi-Fermi liquid.

Self-energy correction.— To implement the outlined plan, we must calculate the self-energy. For definiteness, consider the self-energy for right-movers. Corresponding diagram is shown in Fig. 1.

The expression which must be evaluated is

$$\Sigma_{k,i\nu}^R = -(g')^2 T^2 \sum_{i\Omega, i\nu'} \int_{Q,q} (2q-2k)^2 G_{k-Q,i\nu-i\Omega}^{R,0} \quad (10)$$

$$\times G_{q-Q,i\nu'}^{L,0} G_{q,i\Omega+i\nu'}^{L,0}.$$

In this equation $\int_k \dots = \int (dk/2\pi) \dots$; the free Matsubara propagator is $G_{k,i\omega}^{p,0} = (i\omega - \varepsilon_k^p)^{-1}$, where the fermion dispersion is $\varepsilon_k^p = pv_F k + v_F' k^2$. The factor $(2k-2q)^2$ appears because each interaction line contributes a factor of $g'(2k-2q)$ to the diagram. The overall minus sign accounts for the presence of a single fermion loop. Calculating momentum integrals we assume that

$$|q|, |Q| < \Lambda < k_F = \frac{v_F}{2v_F'}, \quad (11)$$

where k_F is the Fermi momentum. This way we may avoid complications arising from spurious zeros of ε_k^p which are located at $k = -2pk_F$.

Performing the standard summation over $i\Omega$ and $i\nu$ and taking the limit $T \rightarrow 0$ we find

$$\Sigma^R = (g')^2 \int_{Q,q} (2q-2k)^2 [\theta(-\varepsilon_q^L) - \theta(-\varepsilon_{q-Q}^L)] \quad (12)$$

$$\times \frac{\theta(\varepsilon_{q-Q}^L - \varepsilon_q^L) - \theta(\varepsilon_{k-Q}^R)}{i\nu - \varepsilon_{k-Q}^R - \varepsilon_q^L + \varepsilon_{q-Q}^L}.$$

For our purposes it is convenient to evaluate the imaginary part of the retarded self-energy:

$$\text{Im } \Sigma_{\text{ret}}^R = -\pi (g')^2 \int_{Q,q} (2q-2k)^2 [\theta(-\varepsilon_q^L) - \theta(-\varepsilon_{q-Q}^L)] \quad (13)$$

$$\times [\theta(\varepsilon_{q-Q}^L - \varepsilon_q^L) - \theta(\varepsilon_{k-Q}^R)] \delta(\nu - \varepsilon_{k-Q}^R - \varepsilon_q^L + \varepsilon_{q-Q}^L).$$

Now we integrate over Q :

$$\text{Im } \Sigma_{\text{ret}}^R = -(g')^2 \int_q \frac{(k-q)^2}{v_F + v_F'(k-q)} [\theta(q) - \theta(q-Q^*)] \quad (14)$$

$$\times [\theta(\varepsilon_{q-Q^*}^L - \varepsilon_q^L) - \theta(\varepsilon_{k-Q^*}^R)],$$

where $Q^*(q)$ delivers zero to the argument of the delta-function in Eq. (13):

$$Q^* = -\frac{\Delta\nu}{2v_F + 2v_F'(k-q)}, \quad \Delta\nu = \nu - \varepsilon_k^R. \quad (15)$$

Thus, we need to evaluate the integral

$$I = \int_0^{q^*} dq \frac{(k-q)^2 [\theta(\varepsilon_{q-Q^*}^L - \varepsilon_q^L) - \theta(k-Q^*)]}{v_F + v_F'(k-q)}, \quad (16)$$

where upper limit of integration $q^* \approx -\Delta\nu/2v_F$ satisfies the equation $q^* = Q^*(q^*)$. It is easy to check that

$$\theta(\varepsilon_{q-Q^*}^L - \varepsilon_q^L) = \theta(Q^*[v_F - v_F'(2q-Q^*)]) = \theta(Q^*). \quad (17)$$

Further, analyzing Eq. (15), we determine that the sign of Q^* coincides with the sign of $(\varepsilon_k^R - \nu)$. Consequently,

$$\theta(\varepsilon_{q-Q^*}^L - \varepsilon_q^L) = \theta(\varepsilon_k^R - \nu), \quad (18)$$

where the function on the right-hand side is independent of the integration variable q .

The second step-function $\theta(k-Q^*)$ in Eq. (16) can be evaluated easily near the mass surface $\nu = \varepsilon_k^R$. When the mass surface is approached, $Q^* \rightarrow 0$; consequently, $\theta(k-Q^*) = \theta(k)$.

Since both step-functions are independent of the integration variable q , the integral I can be trivially evaluated to the lowest order in $\nu - \varepsilon_k^R$. Keeping the most singular term, we derive

$$\text{Im } \Sigma_{\text{ret}}^R = -\frac{(g'k)^2}{4\pi v_F^2} (\varepsilon_k^R - \nu) [\theta(\varepsilon_k^R - \nu) - \theta(k)] + \delta\Sigma, \quad (19)$$

where $\delta\Sigma$ stands for less singular terms. To obtain $\text{Re } \Sigma_{\text{ret}}^R$ we use Kramers-Kronig relations. For the first term in Eq. (20) the Kramers-Kronig integral can be easily calculated analytically (with Λ playing the role of the high-energy cutoff):

$$\Sigma_{\text{ret}}^R = \frac{(g'k)^2}{4\pi^2 v_F^2} (\nu - \varepsilon_k^R) \ln \left(\frac{\nu - \varepsilon_k^R + i0}{v_F \Lambda} \right) + \dots \quad (20)$$

The less-singular contribution due to $\delta\Sigma$ is replaced by the ellipsis.

Equation (20) resembles Eq. (4): both have singularities at the mass surface. Yet, there is an important difference: expression in Eq. (20) has an extra k^2 factor, which acts to weaken the singular contribution at small k . We will see that peculiar properties of our system may be traced back to this feature of the self-energy.

The quasiparticle residue [43]

$$Z^R(k) = \frac{1}{1 - \partial \Sigma_{\text{ret}}^R / \partial \nu} \Big|_{\nu=\varepsilon_k^R}, \quad (21)$$

$$\frac{\partial \Sigma_{\text{ret}}^R}{\partial \nu} = \frac{(g'k)^2}{4\pi^2 v_F^2} \ln \left(\frac{\nu - \varepsilon_k^R + i0}{v_F \Lambda} \right) + \dots \quad (22)$$

vanishes for any finite k due to the divergence of $\partial \Sigma_{\text{ret}}^R / \partial \nu$ on the mass surface. Thus, like the Tomonaga-Luttinger model, our system does not support the perturbatively-defined quasiparticles. However, since the interaction is irrelevant, in the Matsubara domain $\partial \Sigma^R / \partial \nu$ remains finite, while expression in Eq. (5) diverges. The divergence is sensitive to temperature: if one replaces the step-functions in Eq. (13) by appropriate Fermi functions, the resultant $\text{Im } \Sigma_{\text{ret}}^R$ becomes continuous function of its arguments. Consequently, $\text{Re } \Sigma_{\text{ret}}^R$ becomes finite. Note also, that Eq. (22) does not contain v_F' explicitly. Thus, the destruction of the quasiparticles occurs even for systems with linear dispersion, as long as interaction is non-zero $g' \neq 0$.

Despite the absence of the quasiparticles, the fermionic occupation numbers $n_k^p = \langle c_{pk}^\dagger c_{pk} \rangle$ remain well-defined.

This is not surprising: any finite-order correction to a ground-state matrix element due to irrelevant interaction is finite (since Z_k is a property of an excited state, it is exempt from this rule). To calculate δn_k^p explicitly we start with the formula $n_k^R = -\int_{-\infty}^0 \frac{d\nu}{\pi} \text{Im} G_{\text{ret},k,\nu}^R$. Therefore, the second-order correction is equal to

$$\delta n_k^R = -\int_{-v_F\Lambda}^0 \frac{d\nu}{\pi} \text{Im} \left[\left(G_{\text{ret}}^{R,0} \right)^2 \Sigma_{\text{ret}}^R \right]. \quad (23)$$

Substituting the expressions for $G_{\text{ret}}^{R,0}$ and Σ_{ret}^R it is easy to show that

$$\left(G_{\text{ret}}^{R,0} \right)^2 \Sigma_{\text{ret}}^R = \frac{(g'k)^2}{8\pi^2 v_F^2} \frac{\partial}{\partial \nu} \left[\ln \left(\frac{\nu - \varepsilon_k^R + i0}{v_F\Lambda} \right) \right]^2 + \dots \quad (24)$$

where, as above, the ellipsis stands for the less-singular contributions to Σ_{ret}^R . With the help of this formula the integral in Eq. (23) can be trivially evaluated

$$\delta n_k^R \approx \frac{(g'k)^2}{4\pi^2 v_F^2} \theta(\varepsilon_k^R) \ln \left(\frac{\varepsilon_k^R}{v_F\Lambda} \right) + \dots, \quad (25)$$

which is finite and small for any $|k| < \Lambda$, provided that g' is small.

Quasi-Fermi liquid.— The calculations presented above prove that the quasi-Fermi liquid of 1D spinless fermions constitute a distinct state of matter. Indeed, it is not a Tomonaga-Luttinger liquid: since δn_k^R , Eq. (25), is small, the quasi-Fermi liquid occupation number is discontinuous at the Fermi energy, while the Tomonaga-Luttinger's n_k^p is continuous. [This dissimilarity is a consequence of the fact that the marginal interaction in the Tomonaga-Luttinger Hamiltonian induces stronger singularity of the self-energy diagram than the singularity of Eq. (20). As a result, for the Tomonaga-Luttinger liquid the occupation number correction diverges for small k .]

On the other hand, the state of matter we are dealing with is not a Fermi liquid because it has no perturbatively defined fermionic quasiparticles. (Heuristic non-perturbative construction of excitations for H_{ii} is discussed in Supplemental Material.) However, the system retains certain features of the Fermi liquid: as we have mentioned in the previous paragraph, the occupation number exhibits finite discontinuity at $k = 0$. This discontinuity exists even though the quasiparticles do not.

Let us now discuss experimental identification of the quasi-Fermi liquid. Due to its peculiar nature, the quasi-Fermi liquid may present itself on experiment as an ordinary Fermi liquid, unless the measurements are done at sufficiently high energy. Indeed, formally, the quasiparticle residue diverges for any finite k , however, the divergence becomes progressively weaker as k approaches the Fermi point.

To appreciate the latter point imagine that the single-fermion spectral function is measured, and the quasiparticle residue is extracted. For an experimental apparatus with finite resolution width Ω the measured value of

$\delta Z_k^{R,\Omega}$ is never divergent

$$|\delta Z_k^{R,\Omega}| = \frac{(g'k)^2}{4\pi^2 v_F^2} \ln \left(\frac{v_F\Lambda}{\Omega} \right) < \infty. \quad (26)$$

In this expression the divergence of δZ_k^R , Eq. (22), is cut at the energy scale $\sim \Omega$. Nonetheless, it is possible that $|\delta Z_k^{R,\Omega}| > 1$, provided that k is not too small: $k > k^\times$, where k^\times is equal to

$$k^\times = \frac{2\pi v_F}{g' \sqrt{\ln \left(\frac{v_F\Lambda}{\Omega} \right)}}. \quad (27)$$

The quantity k^\times defines the crossover scale: for momenta smaller than k^\times the behavior of the system is indistinguishable from the usual Fermi liquid. Indeed: $|k| < k^\times \Leftrightarrow |\delta Z_k^{R,\Omega}| < 1$. Thus, the characteristic divergence of the quasiparticle residue may be measured only for momenta k in the interval: $k^\times < |k| < \Lambda$. If the resolution is so poor that $k^\times > \Lambda$, the experimentally measured behavior of the system is indistinguishable from the Fermi liquid for any k . This imposes a restriction on Ω : it has to be smaller than $\Omega_{\text{max}} = v_F\Lambda \exp \left[- (2\pi v_F/g'\Lambda)^2 \right]$. Therefore, unless we have an apparatus with exponentially sharp resolution, phenomenology of the quasi-Fermi liquid may be observed only if g' is not too small. However, at larger g' our perturbation theory becomes less accurate. Can the quasi-Fermi liquid survive in the non-perturbative regime? We hypothesize that the quasi-Fermi liquid, much like the Fermi or Tomonaga-Luttinger liquids, constitutes its own separate universality class, and the quasi-Fermi liquid phenomenology extends beyond the small- g' region.

Cold atoms.— Finally, let us discuss possible implementation of the quasi-Fermi liquid with the help of the cold fermion atoms in a trap [15]. To characterize the gas, instead of using full inter-atomic potential $V(x)$, the interactions in such systems are modeled by an effective delta-function-like potential with the corresponding coupling g . Such formalism is equivalent to our H_{int} [see Eq. (3)], which also describes the contact interaction between the fermions. Experimentally, it is possible to control the magnitude and sign of coupling g . Moreover, g can be nullified. When this nullification occurs, however, the atoms will not behave as non-interacting gas. Indeed, vanishing of H_{int} does not imply the vanishing of the irrelevant H'_{int} , which drives the system toward the quasi-Fermi liquid.

To be more specific, consider the following toy model: a 1D fermions gas with weak interaction $\int dx dx' V(x - x') \rho(x) \rho(x')$. For such a situation the effective low-energy Hamiltonian of the form H , Eq. (8), may be derived. The (bare) coupling constants are:

$$g = 2 \int V(x) [1 - \cos(2k_F x)] dx, \quad (28)$$

$$g' = - \int x V(x) \sin(2k_F x) dx. \quad (29)$$

Usually, it is enough to retain g , and g' is discarded due to its irrelevance.

Imagine now that we adjust V to cancel g . [Strictly speaking, we must eradicate the renormalized coupling $g^{\text{eff}} = g + \mathcal{O}((g')^2)$; however, when V is small, the corrections to the bare coupling are insignificant.] In a generic situation g' remains finite even when $g = 0$. Of course, in this case g' cannot be neglected, and H_{ii} [see Eq. (9)] is realized. The aim of this discussion is to demonstrate that upon destruction of the marginal interaction one does not arrive at the free fermion theory. Rather, the new effective theory has the irrelevant interaction term, and our system becomes the quasi-Fermi liquid.

Conclusions.— To conclude, we have shown that the

system of 1D spinless fermions with the irrelevant interaction is neither a Fermi liquid, nor it is a Tomonaga-Luttinger liquid. Instead, our system constitutes a distinct state of matter, which we propose to call the quasi-Fermi liquid. The generic Tomonaga-Luttinger Hamiltonian with non-linear dispersion is known to be unitary equivalent to the Hamiltonian of such quasi-Fermi liquid. In addition, we speculated that the quasi-Fermi liquid may be realized using the cold atoms in 1D trap.

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Supplemental Material for “One-Dimensional Fermions with neither Luttinger-Liquid nor Fermi-Liquid Behavior”

ABSTRACT

To go beyond the limitations of the second-order perturbation theory, in this Supplementary we discuss our model using non-perturbative three-band Hamiltonian approach. We demonstrate that the hole-like excitations have zero overlap with hole excitations of the non-interacting system. This means that the quasi-Fermi liquid phenomenology survives. On the other hand, if the kinetic energy has the non-linear dispersion term ($v'_F \neq 0$), the particle-like excitations acquire finite lifetime due to the Cherenkov emission process. This formally makes the quasiparticle residue Z finite. However, since in our system the Cherenkov scattering is extremely weak, Z remains small. Thus, we expect that at not too low interaction the quasi-Fermi liquid features may be observed for a particle-like excitation, provided that the excitation momentum is not too small.

I. INTRODUCTION

In the main paper we demonstrated that the second-order correction δZ to the quasiparticle residue Z diverges in our system. Using this observation we argued that for the model under consideration the Landau theory of Fermi liquid is inapplicable. However, one might object, hypothesizing that after an infinite-order resummation a Fermi liquid is recovered. To address this issue we will employ here a non-perturbative approach. It is based on a heuristic mapping of the original Hamiltonian to the so-called three-band Hamiltonian [1]. The latter Hamiltonian can be solved exactly. We will see that for a hole-like excitation, a quasiparticle remains poorly defined even when the treatment is extended beyond the second-order perturbation theory.

As for a particle-like excitations, when $v'_F \neq 0$, the Cherenkov emission of low-lying fermion-hole pairs generates finite lifetime for such excitations. Finite lifetime caps the divergence of δZ , restoring the validity of the perturbation theory. That is, formally, the quasi-Fermi liquid survives for holes, but not for particles. However, we will demonstrate that at not too small interaction and momentum, the suppression of the quasiparticle residue for the particle-like excitations is very strong. Thus, we expect to observe the quasi-Fermi liquid phenomenology for both types of excitations.

II. THREE-BAND HAMILTONIAN FOR A HOLE-LIKE EXCITATION

We begin our discussion with a case of a hole-like excitation. It was argued in Ref. 1. that dynamics of a single hole in a one-dimensional system may be described in terms of the effective three-band Hamiltonian (see figure):

$$H_{3B} = H_{\text{kin}} + H_{\text{int}}, \quad (30)$$

$$H_{\text{kin}} = iv_F \int dx \left(: \psi_L^\dagger \nabla \psi_L : - : \psi_R^\dagger \nabla \psi_R : \right) \quad (31)$$

$$+ \int dx : \psi_h^\dagger (\omega_h - iv_k \nabla) \psi_h :, \quad (32)$$

$$H_{\text{int}} = -\tilde{g}_{\text{hL}} \int dx \rho_L : \psi_h^\dagger \psi_h :.$$

Loosely speaking, this Hamiltonian describes a “high-energy” hole ψ_h with momentum $p \approx k$, which interacts with two bands of low-lying fermionic degrees of freedom ψ_R and ψ_L .

More technically, here ψ_h corresponds to a right-moving hole with momentum p confined within “the high-energy band”:

$$|p - k| < P, \quad 0 < -k \ll \Lambda. \quad (33)$$

The bandwidth is $2P$, where the effective cutoff is chosen to satisfy $0 < P < |k|$. The bare energy of the hole is

$$\omega_h = |\varepsilon_k^{\text{R}}| = |v_F k + v'_F k^2|, \quad (34)$$

its bare velocity is equal to

$$v_k = v_F + 2v'_F k < v_F. \quad (35)$$

Two “low-energy” fields, ψ_R and ψ_L , have their momenta bound according to the inequality (see also figure):

$$|p| < P. \quad (36)$$

The fermion states outside the bands defined by Eqs. (33) and (36) are assumed to be either almost empty, or almost completely occupied, thus, they are “integrated out”.

We assume that, due to its irrelevance, the interaction between the low-lying fermions may be neglected (in Ref. 1 the authors studied a model with the marginal interaction; thus, they had to retain the interaction between ψ_R and ψ_L). At the same time, effective interaction between the hole and the low-lying band is finite for finite k . It is characterized by the coupling constant

$$\tilde{g}_{\text{hL}} = 2g'k + \mathcal{O}(g^2). \quad (37)$$

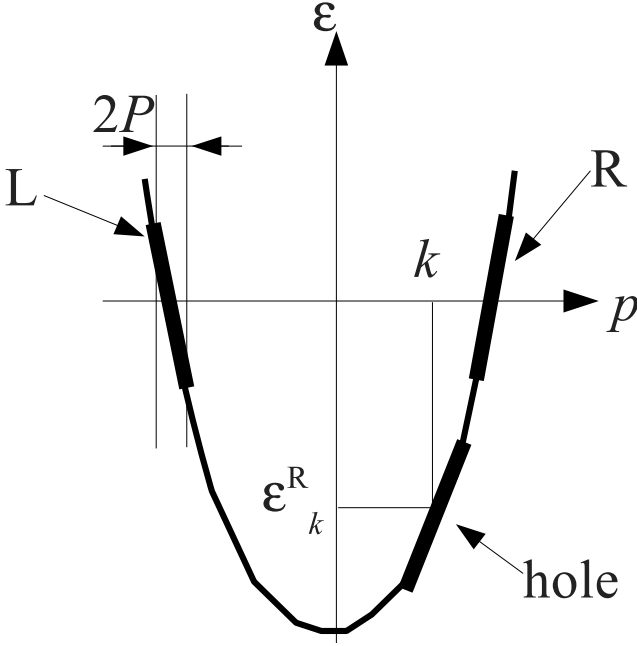


FIG. 2: Kinetic energy of the three-band model. The phase space of the original model is significantly truncated when formulating the three-band model. The bands ('L', 'R', and 'hole') are shown by thick lines. Only the fermion states within these bands are taken into account by the three-band effective Hamiltonian. All other states are "integrated out". The dispersion within the bands is linearized. The "low-energy" left-moving ('L') and right-moving ('R') bands are centered around Fermi energy ($\varepsilon^F = 0$ in our situation), the "high-energy" band in which the hole is located is centered around $\omega_h = \varepsilon_k^R$. The width of these bands is $2P$, where $P < |k|$ serves as a cutoff of the new effective Hamiltonian.

Note that due to irrelevance of the interaction, the coupling constant vanishes when $k \rightarrow 0$. Yet, for any finite k it remains finite. This fact is of cardinal importance for us: scattering of the low-lying excitations by the hole induces the orthogonality catastrophe. That is, the state of the non-interacting system ($\tilde{g}_{hL} = 0$) containing one hole with momentum k has zero overlap with the state of the interacting system ($\tilde{g}_{hL} \neq 0$) in which a quasi-hole with momentum k is created:

$$\langle k, \tilde{g}_{hL} = 0 | k, \tilde{g}_{hL} \neq 0 \rangle = 0. \quad (38)$$

This overlap is related to the quasiparticle residue:

$$Z_k^R = |\langle k, \tilde{g}_{hL} = 0 | k, \tilde{g}_{hL} \neq 0 \rangle|^2. \quad (39)$$

Therefore, the residue vanishes. Clearly, if $Z_k^R = 0$, the perturbation theory fails, which we have demonstrated in the main paper. The nullification of Z also disagrees with the basic assumption of the Landau theory of a Fermi liquid.

To prove Eq. (38) it is convenient to bosonize the low-lying degrees of freedom of the three-band Hamiltonian:

$$H_{\text{kin}} = \frac{v_F}{2} \int dx [:(\nabla\varphi_+)^2: + :(\nabla\varphi_-)^2:] \quad (40)$$

$$+ \int dx \psi_h^\dagger (\omega_h - i v_k \nabla) \psi_h, \\ H_{\text{int}} = -\frac{\tilde{g}_{hL}}{\sqrt{2\pi}} \int dx : \psi_h^\dagger \psi_h : (\nabla\varphi_+), \quad (41)$$

where bosonic fields φ_\pm are related to the chiral densities:

$$\rho_R = -\frac{1}{\sqrt{2\pi}} \nabla\varphi_-, \quad \rho_L = \frac{1}{\sqrt{2\pi}} \nabla\varphi_+. \quad (42)$$

Using the commutation relations

$$[\varphi_\pm(x); \varphi_\pm(y)] = \mp \frac{i}{2} \text{sgn}(x-y), \quad (43)$$

$$[\varphi_+(x); \varphi_-(y)] = 0, \quad (44)$$

we can prove that for the operator W defined as

$$W = \int dx : \psi_h^\dagger \psi_h : \varphi_+, \quad (45)$$

the following relation is valid

$$[H_{\text{kin}}; W] = i \frac{\sqrt{2\pi}(v_F + v_k)}{g_{hL}} H_{\text{int}}. \quad (46)$$

Consequently, the unitary transformation $U_\theta = e^{i\theta W}$ diagonalizes H_{3B} :

$$\overline{H} = U_\theta H_{3B} U_\theta^\dagger = H_{\text{kin}} + \dots, \quad (47)$$

$$\text{provided that } \theta = -\frac{g_{hL}}{\sqrt{2\pi}(v_F + v_k)}. \quad (48)$$

The ellipsis stands for correction to the bare energy of the hole ω_h , which is introduced by $U_\theta H_{\text{int}} U_\theta^\dagger$.

In the ground state of non-interacting Hamiltonian \overline{H} , which we denote by $|0, \tilde{g}_{hL} = 0\rangle$, there is no hole, and all bosonic modes are in their ground states. We are more interested, however, in the state where a hole with momentum k is present:

$$|k, \tilde{g}_{hL} = 0\rangle = \int \frac{dx}{\sqrt{L}} \psi_h^\dagger(x) |0, \tilde{g}_{hL} = 0\rangle. \quad (49)$$

Of course, $|k, \tilde{g}_{hL} = 0\rangle$ is an eigenstate of \overline{H} . The eigenstate of the three-band Hamiltonian H_{3B} with a single hole and momentum k is

$$|k, \tilde{g}_{hL} \neq 0\rangle = U_\theta^\dagger |k, \tilde{g}_{hL} = 0\rangle. \quad (50)$$

Using Eq. (50) we can express the overlap from Eq. (38) as:

$$\langle k, \tilde{g}_{hL} = 0 | k, \tilde{g}_{hL} \neq 0 \rangle = \quad (51)$$

$$\int \frac{dx dx'}{L} \langle 0, \tilde{g}_{hL} = 0 | \psi_h(x) U_\theta^\dagger \psi_h^\dagger(x') | 0, \tilde{g}_{hL} = 0 \rangle.$$

Since $\psi_h^\dagger \psi_h |0, \tilde{g}_{hL} = 0\rangle = 0$, thus, this state is invariant under the transformation U_θ for any θ :

$$U_\theta |0, \tilde{g}_{hL} = 0\rangle = |0, \tilde{g}_{hL} = 0\rangle. \quad (52)$$

This identity and the following expression for the transformed field

$$U_\theta^\dagger \psi_h^\dagger U_\theta = \exp(-i\theta\varphi_+) \psi_h, \quad (53)$$

allow us to write

$$\langle k, \tilde{g}_{hL} = 0 | k, \tilde{g}_{hL} \neq 0 \rangle = \int \frac{dx dx'}{L} \langle \psi_h(x) \psi_h^\dagger(x') \rangle \langle \exp(-i\theta\varphi_+) \rangle, \quad (54)$$

where $\langle \dots \rangle$ stands for expectation value with respect to the non-interacting ground state $|0, \tilde{g}_{hL} = 0\rangle$. Since in the non-interacting system the low-lying bosons and the hole are decoupled, the expectation value decomposes into a product of two matrix elements, one is for the bosonic degrees of freedom, another is for the hole. The fermionic matrix element can be evaluated quite straightforwardly:

$$\begin{aligned} \langle \psi_h(x) \psi_h^\dagger(x') \rangle &= \delta(x - x'), \quad \Rightarrow \\ L^{-1} \int dx dx' \langle \psi_h(x) \psi_h^\dagger(x') \rangle &= 1. \end{aligned} \quad (55)$$

For the bosonic matrix element we derive:

$$\langle \exp(i\theta\varphi_+) \rangle = \exp\left(-\frac{\theta^2}{2} \langle \varphi_+^2 \rangle\right), \quad (56)$$

$$\langle \varphi_+^2 \rangle = \frac{1}{2\pi} \ln(PL) \rightarrow \infty, \quad (57)$$

when $L \rightarrow \infty$. Therefore, $\langle \exp(i\theta\varphi_+) \rangle$ vanishes in the thermodynamic limit, and the orthogonality catastrophe occurs.

Equations (56) and (57) suggest the following interpretation of the orthogonality catastrophe: as a result of scattering off the hole, divergent amount of “soft” fermion-hole pairs are excited, which leads to the nullification of the overlap, Eq. (38), and the quasiparticle residue, Eq. (39). This renders the familiar Fermi liquid theory inapplicable.

Why irrelevant interaction has such dramatic effect on the excited state, but not on the ground state? The irrelevant interaction in the energy domain disappears as the Fermi energy is approached. As a result, its effect on the ground state is amenable to the perturbation theory approach. At the same time, the irrelevant interaction is able to generate non-zero coupling [see Eq. (37)] for any finite- k hole excitation. Due to the irrelevance of H_{int} , the coupling constant \tilde{g}_{hL} vanishes when $k \rightarrow 0$, but it remains finite for any finite k . This coupling is the cause of the orthogonality catastrophe we described above.

III. PARTICLE-LIKE EXCITATION

Thus far, we discussed the hole excitations. Let us now address the case of particle-like excitation. Superficially, one expects that the same orthogonality catastrophe would occur for the particle excitations as well.

This, however, is correct only when $v_F' = 0$. Otherwise, since the group velocity of a particle excitation is higher than the Fermi velocity, it acquires finite lifetime due to Cherenkov emission of particle-hole pairs [2].

The Cherenkov emission is a very weak process in our system: to satisfy momentum and energy conservation laws, two pairs (one right-moving and one left-moving) have to be emitted. The corresponding scattering rate in a model with marginal interaction $g\rho_L\rho_R$ has been evaluated in Ref. 2 [see Eq. (10) in this reference]. It is proportional to the fourth power of the interaction constant and eighth power of k : $\omega_k^{\text{Ch}} \propto (gg''_{\text{RR}})^2 k^8$, where $g''_{\text{RR}} k^2$ characterizes the same-chirality coupling. Such a high power of k is a consequence of the fact that two particle-hole pairs must be emitted.

Since in our system the interaction between fermions of opposing chiralities is irrelevant, we expect that ω_k^{Ch} demonstrates even faster decay. The dimensional analysis suggests that

$$\omega_k^{\text{Ch}} \propto (g'g''_{\text{RR}})^2 k^{10}. \quad (58)$$

The finite scattering rate implies that the self-energy on the mass surface acquires finite imaginary part $\propto \omega_k^{\text{Ch}}$. It caps the divergence of the quasiparticle residue correction δZ . Thus, formally, for a particle-like excitation the Fermi liquid behavior is restored, and the orthogonality catastrophe is avoided. However, due to extreme weakness of the Cherenkov emission, the restoration of the Fermi liquid becomes apparent only in a very narrow region near the mass surface:

$$|\nu - \varepsilon_k^p| \ll \omega_k^{\text{Ch}}. \quad (59)$$

Outside of this area, the quasi-Fermi liquid physics can be observed.

To be more qualitative, let us consider the correction to the quasiparticle residue in the situation $\omega_k^{\text{Ch}} \neq 0$. The correction becomes finite:

$$\begin{aligned} \delta Z_k^{\text{R}} &= \frac{(g'k)^2}{4\pi^2 v_F^2} \ln\left(\frac{v_F \Lambda}{\omega_k^{\text{Ch}}}\right) \\ &= \frac{(g'k)^2}{4\pi^2 v_F^2} \times 10 \ln\left(\frac{\Lambda}{k}\right) + \dots \end{aligned} \quad (60)$$

Neglecting weak logarithmic dependence, we write

$$|\delta Z_k^{\text{R}}| > 10 \delta Z_k^{\text{naive}}, \text{ where } \delta Z_k^{\text{naive}} = \frac{(g'k)^2}{4\pi^2 v_F^2}. \quad (61)$$

That is, while the logarithmic divergence is absent, δZ experiences strong renormalization (one order of magnitude, approximately) as compared to its “naive” estimate $\delta Z_k^{\text{naive}}$. Thus, at not too small g' and k (that is, when $\delta Z_k^{\text{naive}} \lesssim 1$) we can expect significant suppression of the quasiparticle residue for the particle-like excitations. In such a regime, phenomenology of the quasi-Fermi liquid may be observed experimentally.

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